On the Stability and Optimal Decentralized Throughput of CSMA with Multipacket Reception Capability*

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Abstract

With the improvement of the physical layer's ability to receive multiple packets sent simultaneously, the classical collision channel model no longer applies and a general MPR model should be employed. We investigate the effect of MPR on the CSMA protocol, which previously have not been studied under such model. By analyzing the stability region of CSMA, we find that CSMA's open-loop throughput is the same as that of S-ALOHA when signal propagation delay is considered. Then we derive the optimal decentralized closed-loop throughput of CSMA and conclude that carrier sensing can provide throughput gain over S-ALOHA but only if the MPR capability is not sufficiently strong. Essentially carrier sensing can be interpreted as an enhancement over optimal scheduling, both of which become unnecessary for strong physical layers.

I. Introduction

By applying techniques in signal processing, space-time coding and spread spectrum modulation, receivers can separate and correctly decode multiple packets transmitted simultaneously over a wireless channel. This is significantly different from the *classical collision channel* model that practical design and theoretical analysis of many network protocols have usually assumed, namely that a transmitted packet is successfully received only if it does not overlap with another. A case in point is carrier sense multiple access (CSMA) communications [1], which previously have not been studied under the MPR model. In this paper, we investigate the stability of CSMA for such channel model and the corresponding maximum stable throughput (MST) attainable with decentralized control.

Unlike the MPR channel, there have been results reported on the stability and dynamic control of CSMA over the classical collision channel. First, Tobagi and Kleinrock studied this in the last of their series of papers on CSMA [2], then later this was investigated (along with CSMA/CD) by Meditch and Lea in [3] and touched upon by Tobagi and Hunt in [4]. There is also a small section on this subject in Bertsekas and Gallager's

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classic text [5 (Sec. 4.4)]. Although they used different definitions of stochastic stability and attacked the problem with different approaches, they all concluded that CSMA under the classical collision channel is unstable. This is an expected result for random access protocols like ALOHA and CSMA as both have traffic load-throughput curves of the same concave shape [1,5 (Sec. 4.4)].

In here, we extend on their work for CSMA to MPR channels. We employ a different definition of stability and approach from [2] and [3]. We define stability as the one used by Ghez, Verdú and Schwartz's work on slotted ALOHA (S-ALOHA) with MPR [6,7] and use a drift analysis similar to theirs.

II. The Network Model

We consider a network with infinite number of stations contending to transmit data packets to a central base station. Each station has the ability to *carrier sense*, namely to detect whether the channel is currently idle or busy, and we assume the time required to do this is negligible (i.e., that there is zero detection time). We employ a slotted-time system in which transmissions may begin only at the start of a slot and that every station is synchronized. Slot duration is set to be the maximum signal propagation time of $\tau = d_{max}/c$ seconds, where d_{max} is the maximum separation distance between the stations and c is the speed of light. This ensures that after a transmission stops, every station will find the channel to be clear after one slot's time; thus, each transmission must be preceded by an idle slot. Packets are of constant length lasting T seconds, where we assume $T > \tau$. As in [1] and [5 (Sec. 4.4)], without loss of generality, we choose $T = 1$, which is equivalent to expressing time in units of T. We also express the slot duration in this normalized time unit as $\alpha = \tau/T$, $0 < \alpha < 1$. Consequently, a packet lasts for $1/\alpha$ slots, where we assume $1/\alpha$ is an integer (as in [1]).

In our network model, packet arrival statistics is independent and identically distributed from slot to slot. Denote A_k to be the number of new packets arriving during slot k and

$$P[A_k = n] = \lambda_n \quad (n \geq 0),$$

such that the mean arrival rate per slot is

$$\lambda_{\text{slot}} = \sum_{n=1}^{\infty} n\lambda_n = \alpha\lambda,$$

where λ is the mean arrival rate per normalized time unit (i.e., per packet duration) and is finite. By expressing the arrival statistics in this manner, λ becomes equivalent to that considered in [1,6,7], facilitating us with meaningful throughput comparisons with S-ALOHA with MPR and existing results on CSMA over the collision channel.

We employ the symmetric MPR channel model introduced in [6,7], of which the successful reception probabilities depend only on the number of packets transmitted in the slot. Given that n packets are transmitted, for $1 \leq n \leq \infty$, $0 \leq k \leq n$, let

$$C_{n,k} = P[k \text{ packets are correctly received} |\ n \text{ are transmitted}] \tag{1}$$

and the channel model can be succinctly characterized by the multipacket reception matrix:

$$\mathbf{C} = \begin{pmatrix} C_{1,0} & C_{1,1} & & \\ C_{2,0} & C_{2,1} & C_{2,2} & \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}. \tag{2}$$

The symmetric MPR channel model is a generalized formulation and embodies as a special case also the classical collision channel, which has MPR matrix

$$\mathbf{C} = \begin{pmatrix} 0 & 1 & & & \\ 1 & 0 & 0 & & \\ \vdots & \vdots & \vdots & \ddots & \end{pmatrix}. \tag{3}$$

We denote the expected number of packets correctly received with a collision set of n stations by

$$\mathcal{C}_n \triangleq \sum_{k=1}^{n} k C_{n,k}, \tag{4}$$

and assume that its limit $\mathcal{C} = \lim_{n\to\infty} \mathcal{C}_n$ exists, as usually is for practical cases [7].

In this paper, we focus on the immediate first transmission (IFT) policy: When a new packet arrives at an inactive station, it will sense the channel at the start of the next slot, and if the channel is idle, the station transmits the packet. If the channel is sensed busy (i.e., that there is at least one other station currently transmitting), then the station issues a random backoff delay after which it reattempts transmission; this process is repeated if the channel is subsequently found busy or that the packet is not successfully received. We see that this policy is exactly that of the *slotted non-persistent CSMA* protocol introduced in [1]. Also, each station's random backoff is modelled as independent sampling from a geometric distribution with parameter p; in other words for a given slot each station will attempt transmission with probability p or defer with probability $1-p$. Moreover, we assume each station becomes aware of their transmission outcome immediately, whether it is successful or not, and without expending extra cost; in this regard, acknowledgements introduce only fixed overheads and hence can be neglected from the model without affecting the analysis and comparisons.

III. Ergodicity Region

We define a network to be stable if the Markov chain, $\{X_t\}_{t\geq 0}$, that represents the number of backlogged packets in the system is ergodic and unstable otherwise. For our network model, we define the Markov chain such that state transitions occur after either an idle slot or an idle slot followed by a transmission. To facilitate our discussion, we will refer to either of these events as a *transmission period* (TP).

It is straightforward to see that $\{X_t\}$ is a discrete-time homogeneous Markov chain. Its transition probabilities can also be readily computed:

$$P_{0,0} = \lambda_0 + \sum_{n=1}^{\infty} \lambda_n C_{n,n} \Lambda_0, \tag{5}$$

$$P_{0,k} = \sum_{n=1}^{\infty} \lambda_n \sum_{s=\max(0,n-k)}^{n} C_{n,s} \Lambda_{k-(n-s)} \qquad (k \geq 1), \tag{6}$$

and for $i \geq 1$,

$$P_{i,i-k} = \sum_{n=0}^{\infty} \lambda_n \sum_{j=k}^{i} B_i(j) \sum_{s=k}^{j} C_{n+j,n+s} \Lambda_{s-k} \qquad (1 \leq k < i), \tag{7}$$

$$P_{i,i} = \lambda_0 B_i(0) + \sum_{n=0}^{\infty} \lambda_n \sum_{j=0}^{i} B_i(j) \sum_{s=0}^{j} C_{n+j,n+s} \Lambda_s, \tag{8}$$

$$P_{i,i+k} = \sum_{n=0}^{\infty} \lambda_n \sum_{j=k}^{i} B_i(j) \sum_{s=\max(0,n-k)}^{n+j} C_{n+j,s} \Lambda_{k-(n-s)} \qquad (k \geq 1), \tag{9}$$

where $B_i(j) = \binom{i}{j} p^j (1-p)^{i-j}$ and Λ_k is the probability that there will be exactly k ($k \geq 0$) new arrivals during the $1/\alpha$ slots (eg. $\Lambda_0 = \lambda_0^{1/\alpha}$). We are not concerned with the exact expression for Λ_k as there is no need to analyze these transition probabilities, since doing so would not provide us with much insights on the system. Instead we base our derivation on drift analysis techniques. Also, $\{X_t\}$ will be irreducible and aperiodic as long as it satisfies the sufficient condition: $0 < \lambda_0 < 1$, which we assume holds as it is true for all reasonable scenarios.

The expected drift of the Markov chain at state $n, n \geq 0$, is given by

$$\begin{aligned} d_n &= E[X_{t+1} - X_t | X_t = n] & (10)\\ &= E[\widetilde{A}_t - \Sigma_t | X_t = n], & (11) \end{aligned}$$

where $\widetilde{A}_t$ and Σ_t are respectively the number of new packets that arrived and the number of successful transmissions during transmission period t.

The expected number of arrivals during a TP depends on whether there are arrivals during the idle slot and whether any of the backlogged stations decide to transmit. If there is at least one arrival in the idle slot, then the TP will consist of the idle slot followed by a transmission; this occurs regardless of the number of backlogged stations that transmit. And of course, this situation also occurs even if the idle slot has no packet arrivals but at least one of the n backlogged stations transmits, which occurs with probability $1 - (1-p)^n$. Because the expected number of arrivals during a slot and during a packet transmission are respectively $\alpha\lambda$ and λ, we have

$$\begin{aligned} E[\widetilde{A}_t | X_t = n] &= \alpha\lambda + (1-\lambda_0)\lambda + \lambda_0(1-(1-p)^n)\lambda & (12)\\ &= \lambda[1 + \alpha - \lambda_0(1-p)^n]. & (13) \end{aligned}$$

To find Σ_t, observe that those stations with new packets arriving during the slots of a transmission will find the channel busy and refrain from transmitting until the next TP. Thus, the packets contributing to channel contention are only those that arrived during the idle slot and those that are backlogged at the start of the TP. Similar to the derivation in [6], let R_t be the number of retransmissions in TP t and $\widehat{A}_t$ be the number of arrivals during the idle slot of TP t, then under the MPR channel,

$$P[\Sigma_t = k | X_t = n, \widehat{A}_t = i, R_t = j] = C_{i+j,k}, \tag{14}$$

for $0 \leq j \leq n, 0 \leq k \leq i+j$ and with the convention that $C_{0,0} = 0$,

$$E[\Sigma_t | X_t = n, \widehat{A}_t = i, R_t = j] = \mathcal{C}_{i+j}, \tag{15}$$

and

$$E[\Sigma_t | X_t = n] = \sum_{i=0}^{\infty} \lambda_i \sum_{j=0}^{n} \binom{n}{j} p^j (1-p)^{n-j} \mathcal{C}_{i+j}. \tag{16}$$

Therefore, the expected drift is

$$d_n = \lambda(1+\alpha) - \lambda\lambda_0(1-p)^n - \sum_{i=0}^{\infty}\lambda_i \sum_{j=0}^{n}\binom{n}{j}p^j(1-p)^{n-j}\mathcal{C}_{i+j}. \tag{17}$$

With this and applying a result reported by Ghez et al in [6], we arrive with the following theorem on the ergodicity region of CSMA with MPR.

Theorem 1. *If $\mathcal{C}_n$ has a limit $\mathcal{C} = \lim_{n\to\infty}\mathcal{C}_n$, then a CSMA system using IFT is stable for all arrival distributions such that $\lambda < \frac{1}{1+\alpha}\mathcal{C}$ and is unstable for $\lambda > \frac{1}{1+\alpha}\mathcal{C}$. This also holds if $\mathcal{C}$ is infinite: if $\lim_{n\to\infty}\mathcal{C}_n = +\infty$, then the system is always stable.*

Proof. From (17) we can see that $|d_n| \le 4\lambda + np$ and so it is finite. Using Lemma 1 of [6], which states that

$$\lim_{n\to\infty}\sum_{i=0}^{\infty}\lambda_i\sum_{j=0}^{n}\binom{n}{j}p^j(1-p)^{n-j}\mathcal{C}_{i+j} = \mathcal{C},$$

we see that the limit of equation (17) is

$$\lim_{n\to\infty} d_n = \lambda(1+\alpha) - \mathcal{C}. \tag{18}$$

Then by Pakes' Lemma [8, Theorem 2] our result on the stable region follows.

To obtain our result on the unstable region, we will verify that Kaplan's condition [9] is satisfied provided that $\mathcal{C}_n < L$, $n \ge 1$, for some $L \in (0,\infty)$. According to [10], this is equivalent to showing that the downward part of the drift, given by $D(i) = -\sum_{k=1}^{i} kP_{i,i-k}$, is bounded below.

$$\begin{aligned}
D(i) = -\sum_{k=1}^{i} kP_{i,i-k} &= -\sum_{k=1}^{i} k\sum_{n=0}^{\infty}\lambda_n\sum_{j=k}^{i}B_i(j)\sum_{s=k}^{j}C_{n+j,n+s}\Lambda_{s-k} \\
&= -\sum_{j=1}^{i}B_i(j)\sum_{n=0}^{\infty}\lambda_n\sum_{s=1}^{j}\sum_{k=1}^{s}kC_{n+j,n+s}\Lambda_{s-k} \\
&> -\sum_{j=1}^{i}B_i(j)\sum_{n=0}^{\infty}\lambda_n\sum_{s=1}^{j}sC_{n+j,n+s}\sum_{k=1}^{s}\Lambda_{s-k} \\
&> -\sum_{j=1}^{i}B_i(j)\sum_{n=0}^{\infty}\lambda_n\mathcal{C}_{n+j} \\
&> -L
\end{aligned}$$

□

As $\mathcal{C} = 0$ for the collision channel, we arrive with yet another result that confirms CSMA over the collision channel is inherently unstable. Furthermore, since the stability for S-ALOHA is achieved as long as $\lambda < \mathcal{C}$ [6], Theorem 1 implies that for fixed or uncontrolled transmission probability—what is referred to as *open-loop control*, CSMA has a lower MST than that of S-ALOHA under the same MPR channel. But of course we are actually treating CSMA unfairly by assuming the signal propagation delay has no effect on S-ALOHA. For if one were to implement it in practice, propagation delay would in fact destroy the synchronous assumption and cause potentially harmful interference between the packets. As originally suggested for S-ALOHA [11], we should avoid this dilemma by designing the slot size to be $1+\alpha$ time units, so that each transmission is completely propagated throughout the network before a new one begins. Thus for a fair comparison with CSMA, we should consider such a version of S-ALOHA. Via the same method used in [6], we can easily derive that S-ALOHA with propagation delay is stable for $\lambda < \frac{1}{1+\alpha}\mathcal{C}$. Therefore, we conclude that the two protocols' open-loop throughputs are the same.

IV. Optimal Decentralized Throughput

Like for S-ALOHA, there have been decentralized strategies reported that stabilize CSMA over the collision channel [2,3]. If we apply these strategies or those that have been shown for random access in general to CSMA with MPR, we should expect a better resulting throughput than the open-loop throughput, denoted by $\eta_{o,c}$, discussed above.

Similar to the decentralized schemes analyzed in [7], [12], [13] and the references therein, we consider schemes in which stations adjust their retransmission probability according to the channel feedback. These schemes can be characterized in this form [7],

$$p_t = F(S_t) \qquad S_{t+1} = G(S_t, Z_t), \tag{19}$$

where p_t is the retransmission probability for TP t, S_t is an estimate of the backlog X_t at the beginning of TP t, and Z_t is the feedback at the end of TP t. We represent this system by a two-dimensional discrete-time homogeneous Markov chain, $\{X_t, S_t\}_{t\geq 0}$. It will also be irreducible and aperiodic if the same sufficient condition stated for $\{X_t\}$ previously is satisfied, which we also assume holds here as well.

Toward finding the optimal decentralized throughput for CSMA with MPR, we first study the case in which the stations have perfect state information, i.e., that each station knows X_t at the beginning of TP t, and consider control algorithms of which stations adjust their retransmission probability according to X_t. Therefore, we can proceed by analyzing the same one-dimensional Markov chain in Section III but with $p = p_t = F(X_t)$.

Certainly, such a system would require some form of central controller to convey the value of X_t to each station. But under this ideal case, we will obtain the upper bound on the optimal throughput achievable by decentralized control schemes and, most importantly, determine the maximum improvement, if any, that carrier sensing has upon S-ALOHA over MPR channels. Subsequently, we could see the conditions under which implementing carrier sensing becomes worthwhile. We would also gain helpful insights for designing true decentralized schemes, like those that estimate X_t from the channel feedback.

Theorem 2. *There exists a retransmission probability p_n^* that minimizes the expected drift at d_n and with which the CSMA system is stable for $\lambda < \eta_{c,c}$ and unstable for $\lambda > \eta_{c,c}$, where*

$$\eta_{c,c} = \sup\left\{\lambda : \lambda < \frac{1}{1+\alpha}\sup_{x\geq 0} e^{-x}\left(\lambda_0\lambda + \sum_{n=0}^{\infty}\frac{x^n}{n!}\sum_{j=0}^{\infty}\lambda_j\mathcal{C}_{n+j}\right)\right\}. \tag{20}$$

Proof. By defining $y_n(p) = \lambda\lambda_0(1-p)^n + \sum_{i=0}^{\infty}\lambda_i\sum_{j=0}^{n}\binom{n}{j}p^j(1-p)^{n-j}\mathcal{C}_{i+j}$, we can write the expected drift equation (17) as $d_n(p) = \lambda(1+\alpha) - y_n(p)$. Since $y_n(p)$ attains a global maximum on $[0,1]$, there exists

$$p_n^* = \arg\max_{p\in[0,1]} y_n(p) = \arg\min_{p\in[0,1]} d_n(p); \tag{21}$$

in other words, there exists a retransmission probability p_n^* that minimizes the drift d_n at state n.

Following the same steps used in the proof of Theorem 1 of [7], we can show that $y_n(x/n)$ converges uniformly to $y(x)$ for $x \geq 0$ with

$$y(x) = e^{-x}\left(\lambda_0\lambda + \sum_{n=0}^{\infty}\frac{x^n}{n!}\sum_{j=0}^{\infty}\lambda_j\mathcal{C}_{n+j}\right). \tag{22}$$

So it follows that

$$\lim_{n\to\infty} y_n(p_n^*) = \lim_{n\to\infty}\sup_{x\geq 0} y_n(x/n) = \sup_{x\geq 0} y(x). \tag{23}$$

Then by applying Pakes' Lemma [8, Theorem 2] the stable region follows.

For the unstable region, we can proceed in the same fashion as we did in Theorem 1 above but substituting $p_t = F(X_t)$ for p. Since just like p, $p_t \in [0,1]$, thus the result obtained there with p_t is unchanged. □

Although it is intuitively clear that the system is unstable with any control as long as $\lambda > \eta_{c,c}$, but as pointed out in [7], it is still necessary to verify that the two-dimensional Markov chain $\{X_t, S_t\}$ is non-ergodic under such a condition too. For otherwise there might exist a decentralized scheme of the form of (19) that is stable with $\lambda > \eta_{c,c}$.

Theorem 3. *The best throughput achievable by any decentralized control policy in the form of* (19) *is* $\eta_{c,c}$.

Proof. The proof is essentially the same as that for Theorem 2 of [7], which gives the analogue result for decentralized control in S-ALOHA. Specifically, we employ results from [14] that generalize Kaplan's result for multidimensional Markov chains to show $\{X_t, S_t\}$ is non-ergodic if $\lambda > \eta_{c,c}$. Consider the Lyapunov function $V(n,s) = n$ for Markov chain $\{X_t, S_t\}$. Suppose $\lambda > \eta_{c,c}$. Then

$$\begin{aligned} E[V(X_{t+1},S_{t+1}) - V(X_t,S_t)|X_t = n, S_t = s] &= \lambda(1+\alpha) - y_n(F(s)) \\ &\geq d_n(p_n^*) \\ &= (1+\alpha)(\lambda - \eta_{c,c}) \end{aligned} \tag{24}$$

for all n large enough and all s. This implies that outside of a finite subset of the state space, the drift of V is strictly positive. Therefore, if $\{X_t, S_t\}$ satisfies the generalized Kaplan's condition, then we can conclude that $\{X_t, S_t\}$ is non-ergodic. According to [14], this is equivalent to verifying that the downward part of the drift of V, denoted by $D_V(x)$, is bounded below for all x, with $D_V(x) = \sum_{V(y)-V(x)} P_{xy}(V(y) - V(x))$ and x, y being states in the state space. Let state x be some state $\{n, s\}$.

$$\begin{aligned} D_V(x) &= -\sum_{k=1}^{i} k \sum_z P[X_{t+1} = n-k, S_{t+1} = z | X_t = n, S_t = s] \\ &= -\sum_{k=1}^{i} kP[X_{t+1} = n-k | X_t = n, S_t = s]. \end{aligned} \tag{25}$$

But this is the same as $D(n)$, the drift of the one-dimensional $\{X_t\}$ at state n, given in the proof of our Theorem 1 above, with $p = p_t = F(S_t)$ and $p_t \in (0, 1)$. Therefore, provided that $\mathcal{C}_n < L$, $n \geq 1$, for some $L \in (0, \infty)$, $D_V(x)$ is bounded below. $\square$

The optimal decentralized throughput given by (20) is also referred to as the maximum *closed-loop throughput* achievable by CSMA. Note also that we can easily obtain all the analogue results of stabilized S-ALOHA given in [7] for the case with signal propagation delay via the same methods they used and in particular its optimal decentralized throughput is simply

$$\eta_{c,s} = \frac{1}{1+\alpha} \sup_{x \geq 0} e^{-x} \sum_{n=1}^{\infty} \mathcal{C}_n \frac{x^n}{n!}. \tag{26}$$

Because the solution to $\lambda(1+\alpha) < \sup_{x \geq 0} y(x)$ is given as an implicit equation of λ_n ($n \geq 0$), we cannot proceed further with the analysis if the complete packet arrival distribution is not specified. In particular, we cannot draw any general conclusions on whether stabilized CSMA can do any better than that of S-ALOHA. In order to obtain a meaningful comparison between the two, we proceed by assuming the packet arrival is Poisson distributed, the de facto assumption for modelling networks with infinite population. With this, $y(x)$ is greatly simplified to

$$\begin{aligned} y(x) &= e^{-(x+\alpha\lambda)} \left(\lambda + \sum_{n=0}^{\infty} \frac{x^n}{n!} \sum_{j=0}^{\infty} \mathcal{C}_{n+j} \frac{(\alpha\lambda)^j}{j!} \right) \\ &= e^{-(x+\alpha\lambda)} \left(\lambda + \sum_{n=1}^{\infty} \mathcal{C}_n \frac{(x+\alpha\lambda)^n}{n!} \right), \end{aligned} \tag{27}$$

and computation for $\eta_{c,c}$ becomes much less complicated.

For example, under the collision channel model, $y(x) = e^{-(x+\alpha\lambda)}[x + \lambda(1+\alpha)]$ and $\sup_{x \geq 0} y(x)$ is attained with $x = 1 - \lambda(1+\alpha)$ and (20) reduces to the transcendental equation

$$\eta_{c,c} = \sup\{\lambda : \lambda(1+\alpha) < e^{\lambda-1}, \lambda > 1\}. \tag{28}$$

Solving numerically for $\alpha = 0.01$, we can find that $\eta_{c,c} = 0.865$, which is equal to the value calculated with the throughput expression reported by Kleinrock and Tobagi for slotted non-persistent CSMA (equation 9 of [1]). They also revealed that as $\alpha \to 0$, CSMA could approach perfect (collision) channel utilization, which can also be shown with (28).

Moreover, with Poisson-distributed packet arrival S-ALOHA's closed-loop throughput remains unchanged and is the same for IFT and DFT (delayed first transmission) policies. Then, as the proof of Theorem 3 in [7] indicates that

$$\sup_{x \geq 0} e^{-(x+\alpha\lambda)} \sum_{n=1}^{\infty} \mathcal{C}_n \frac{(x+\alpha\lambda)^n}{n!} = \sup_{x \geq 0} e^{-x} \sum_{n=1}^{\infty} \mathcal{C}_n \frac{x^n}{n!}, \tag{29}$$

to study the throughput improvement that CSMA has over S-ALOHA is equivalent to investigating the effects on the supremum given by (26) with the addition of the term $\lambda e^{-(x+\alpha\lambda)}$. In fact, it is straightforward to see that it is this term that provides the additional throughput gain with carrier sensing.

In general, we find that the amount of throughput improvement depends solely on the nature of the sequence $\mathcal{C}_n$. Specifically, the larger the value of $\bar{n} = \arg\sup \mathcal{C}_n$, the closer

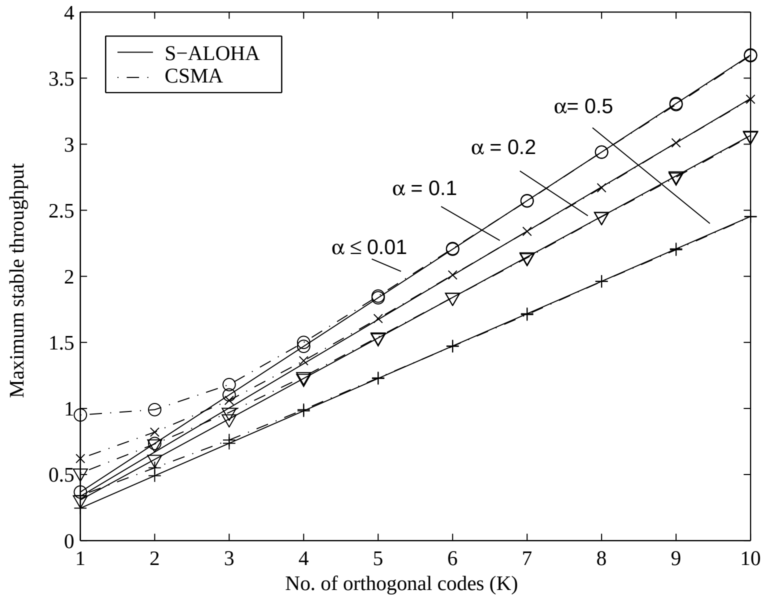

Figure 1: Optimal decentralized stable throughput for orthogonal spreading code networks with CSMA and S-ALOHA.

the value of $\bar{x} = \arg\sup y(x)$ is to $\bar{n}$ (with $\bar{x} \leq \bar{n}$ for all $\mathcal{C}_n$). But if x is large, contribution from the term $\lambda e^{-(x+\alpha\lambda)}$ will be small and consequently CSMA's throughput gain will diminish. Since the sequence $\mathcal{C}_n$ characterizes the MPR capability of the corresponding physical layer, the magnitude of its $\bar{n}$ can be interpreted as a measure of its MPR strength. Therefore, we can conclude that for physical layers that are sufficiently strong, CSMA's optimal decentralized throughput will be near to that of S-ALOHA.

To illustrate this, consider a physical layer consisting of K orthogonal spreading codes with which the stations pick one at random to transmit their packets. A station's packet is decoded successfully at the receiver if and only if no other stations choose the same code. This scenario is equivalent to the frequency-hopping network described in [6] and its closed loop throughput is reported in [7]. We plot in Fig. 1a the optimal decentralized throughput for CSMA and S-ALOHA with various propagation delays. We can see that the extra throughput gained with carrier sensing diminishes when $K > 5$ for all α. The plot also shows the aggravating effects of propagation delay, α. Not only is this convergence point lower with longer propagation delays, the amount of throughput improvement in the weak MPR regime is also smaller. And quite naturally, the overall throughputs are lower with larger α.

Obviously, our finding indicates that the stronger the physical layer is, the lesser we need carrier sensing. It turns out that this interpretation resembles the conclusion made in [15] that under such condition, it becomes unnecessary for scheduling in a multiaccess channel. And actually this is one of the reasons behind our conclusion. The only difference in channel utilization between CSMA and S-ALOHA is that CSMA has idle slot of duration α while that of S-ALOHA lasts the whole packet length (cf. [5, Sec. 4.4]). This savings in overall idle channel time is what contributes to CSMA's throughput gain over S-ALOHA. However, as MPR capability improves and stations transmit more readily, the idle channel time diminishes. Since other than that the success and collision periods on the channel remain the same for the two protocols, so will be for their throughputs. Therefore, in a way, carrier sensing can be thought of as an enhancement on top of optimal scheduling, both of which become extraneous if the physical layer is strong.

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